Maximum time: 45 *minutes*

KEM3 - Maths Olympiad for Std $9^{th}, 10^{th}$ & 11^{th} together @ *ABIMS*

Try on your own ! Don't use calculators ! Think and Answer !

Name:_____ Standard:_____

I Answer the following questions accordingly !

1. $ABCD$ is a parallelogram where $AC = 22$ *unit*; $BD = 14$ *unit*.

   The perimeter of parallelogram $ABCD$ is 52 *unit*.

   The difference between the adjacent sides of the parallelogram $ABCD$ is _____ *unit*.

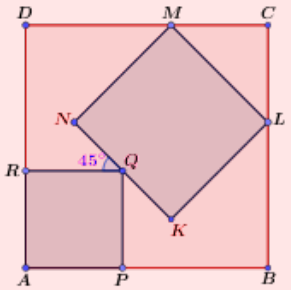
2. x, y, z are real numbers such that $\begin{cases} x+y+z=0 \\ xy+yz+zx=-3 \end{cases}$.

   Then the value of the expression $x^3y+y^3z+z^3x$ is _____.

3. $ABCD$ is a square of side length 35 *unit*; P, L, M, R are points on sides AB, BC, CD, DA respectively;

   K, Q, N are interior points of square $ABCD$ such that $APQR$ and $KLMN$ are squares, as shown.

   Note: Points K, Q, N are collinear points. $\angle RQN = 45°$.



   If the area of square $APQR = 196$ *sq.unit*, then the area of the square $KLMN$, is _____ *sq. unit*.

4. The number of ordered integer pairs (x, y) satisfying the equation $\frac{1}{x} + \frac{2024}{xy} = 2$, is _____.

5. N is a positive integer such that it can be expressed as $a^2+b^2+c^2+d^2$, where a,b,c,d are positive integers. If the number of such ordered quadruples (a,b,c,d) is 43, then which of the following is definitely true?

   A. N is the square of an odd number.

   B. N is the cube of a positive integer.

   C. N is the square of an even number.

   D. none of these.

Maximum time: 120 *minutes*

Number Theory, Algebra, Geometry, Combinatorics

KEM3 - Maths Olympiad for Std 9,10,11 together @ *ABIMS*

Try on your own ! Don't use calculators ! Think and Answer !

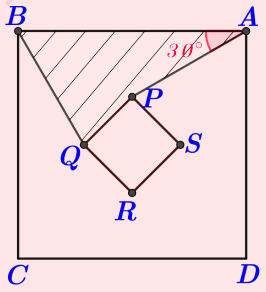
Name:_____ Standard:_____

I Answer the following questions accordingly !

1. How many positive integers from 1 to 2024 are multiples of 3 or 4 but not 12? _____.

2. If the roots of $x^3 - 5x^2 + qx - 8 = 0$ are a, ab, ab^2 such that a, b, q are constants and ab being a real number, then the value of q is _____.

3. $ABCD$ is a square of side length 10 *unit*. P, Q, R, S are interior points of $ABCD$ such that $PQRS$ is a square of side length 4 *unit*. $\angle BAP = 30°$, as shown.
   Squares $ABCD, PQRS$ share a common centre.



   Note: Figure is not drawn to scale.
   The area of the shaded quadrilateral, is _____ *sq.unit*.

4. The $H.C.F$ and the $L.C.M$ of two positive integers are 18 and 32400 respectively.
   If the sum of the integers is 3762, what is their difference? _____.

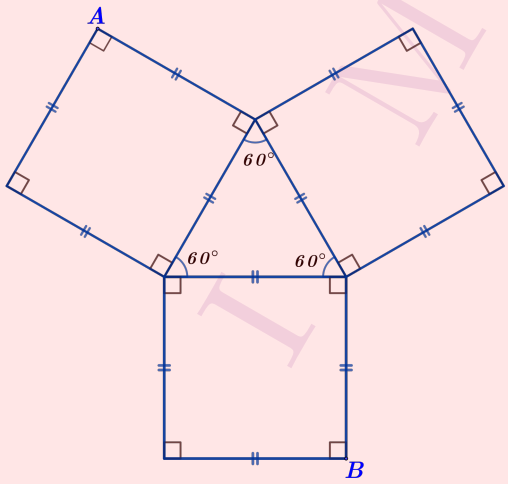
5. How many positive integers n that are perfect cubes that exactly divides $3! \times 5! \times 7! \times 9!$? _____.
   Note: $k!$ called as factorial of natural number k, is the product of the first k natural numbers.

6. x is a negative integer and y is a positive integer satisfying the equations:

$$\begin{cases} x + y + \dfrac{x}{y} = \dfrac{7}{5} \\ (x+y) \cdot \dfrac{x}{y} = \dfrac{-6}{5} \end{cases}$$

Then, the value of x is _____.

7. On each side of an equilateral triangle, squares are constructed, as shown.



Note: Figure is not drawn to scale.

If the side length of equilateral triangle is 4 unit and the distance between the farthest corners A and B is $m + \sqrt{n}$ *unit* where m, n are positive integers (n is not a perfect square), then $m + n =$ _____.

8. Starting from a two digit prime number, at every step the following operations are performed.

- Multiply the digits of the number.
- Add 4 to the product.

This Process continues with every result obtained. After the third step, 28 is obtained.

$$m \longrightarrow n \longrightarrow p \longrightarrow 28.$$

Here n is not a prime.

What is the starting two digit prime number m? _____.

9. How many 3− digit numbers satisfy the property that the middle digit is the average of the first and the last digits? _____.

10. a, b, c, d, e, f, g are positive integers satisfying the system of 7 equations:

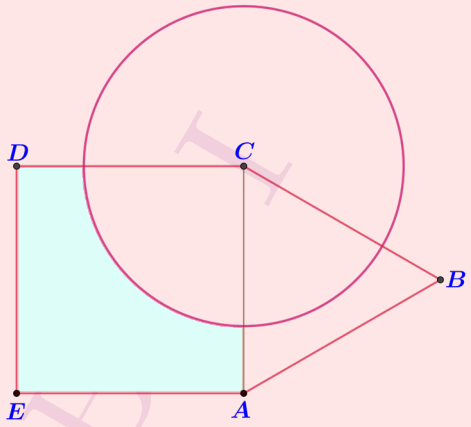
$$\begin{aligned} a+b &= 102 \\ b+c &= 91 \\ c+d &= 103 \\ d+e &= 110 \end{aligned} \qquad \begin{aligned} e+f &= 79 \\ f+g &= 106 \\ g-b &= 7 \end{aligned}$$

How many among a, b, c, d, e, f, g are primes?

A. 3 B. 7 C. 5 D. 0

11. $ABCDE$ is a pentagon such that $ACDE$ is a square and ABC is an equilateral triangle (not overlapping). A circle centered at C has area 24 sq. unit.

The circle intersects 50% of pentagon and the regions of equilateral and the square partly, as shown.



Note: Figure is not drawn to scale.

If the area of the region of the square outside the circle is $\dfrac{a-b\sqrt{3}}{13}$ *sq. unit*, where a, b are integers, then $a+b =$ _____.

12. The single digit numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are placed along the circle in an arbitrary order.

Reading clockwise three consecutive digits, you get a $3-$ digit number.

There are 9 such $3-$ digit numbers altogether.

What is their sum? _____.

13. Let S be the union of the first 2024 terms of each of the following $A.P.'s$(Arithmetic Progressions)

$$1, 4, 7, 10, 13....$$

$$9, 16, 23, 30, 37, ...$$

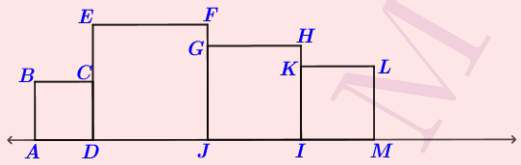
How many distinct numbers are there in set S? _____.

14. a, b, c, d are real numbers satisfying the system of equations

$$\begin{aligned} a+b-c-d &= 0 \\ ab-2bc+cd-2da &= 21 \\ 2ab-bc+2cd-da &= -24 \end{aligned}$$

Then, the value of $7(a-c)^2+17(b-d)^2 =$ _____.

15. In the given figure , $ABCD, DEFJ, GHIJ, IKLM$ are squares. A, D, J, I, M all lie on a straight line, as shown.



If $AM = 20\ unit, CE = 5\ unit, GF = 2\ unit, HK = 1\ unit$, then the sum of the areas of these four squares is _____ $sq.unit$.

16. The smallest $7-$ digit palindrome number having only even digits, is exactly divisible by 7.

The number is _____.

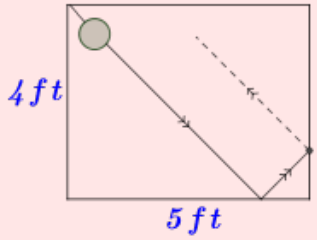
17. How many $4-$ digit positive integers have at least one digit that is a 5 or a 7? _____.

18. p, q are real numbers such that they satisfy the system of equations:

$$\begin{aligned} 25p^2-40q &= -23 \\ 25q^2-30p &= -2 \end{aligned}$$

Then, the value of $50p^2+25q^2$ is _____.

19. A cue ball is shot at a 45 degree angle from the upper left corner of a rectangular billiard table with dimensions 4 ft by 5 ft, as shown,



Note: Assume the ball is perfectly reflected at every bounce.

How many times does the ball bounce before hitting an other corner?_____.

20. A $4-$ digit number has the following properties:

    - It leaves a remainder 8 when divided by 9.
    - It leaves remainder 14 when divided by 15.
    - It has exactly two prime factors both ending with the same digit.

    The smallest such number is _____.

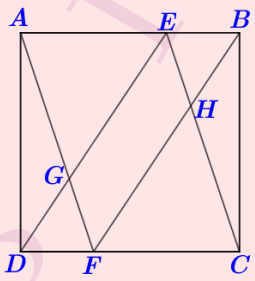
21. How many distinct $8-$ digit numbers are divisible by 9 and have 2024 as their last four digits? _____.

22. If $x(x-y)+y(y-z)+z(z-x)=121$, then the value of

$$(x-y)(y-z)+(y-z)(z-x)+(z-x)(x-y)$$

    is _____.

23. $ABCD$ is a square of area 81 $sq.unit$. E, F are points on AB, CD respectively such that $AE = 6$ $unit$, $DF = 3$ $unit$. AF, DE intersects at point G and BF, CE intersects at points H, as shown.



    Note: Figure is not drawn to scale.

    Then, the area of quadrilateral $EFGH$ is _____$sq.unit$.

24. M, U, L, T, I, P, L, Y are distinct integers such that

$$M \times U \times L \times T \times I \times P \times L \times Y = 310310$$

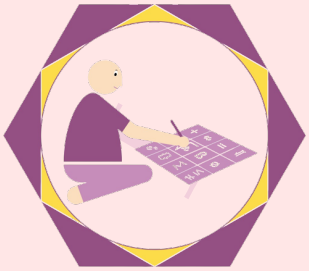
    The least sum of $M+U+L+T+I+P+L+Y =$ _____.

ALGEBRA,NUMBER THEORY, GEOMETRY & COMBINATORICS

ASSIGNMENT - 2

KEM3 - Maths Olympiad for Std $9^{th}, 10^{th}$ and 11^{th} together @ $ABIMS$

Shri Sadagopan Rajesh



Try on your own!

Post your detailed (not just numbers) and neat solution(s) to : kem2022.23@gmail.com (only!)

Mention your Name, Standard INSIDE ANSWER SHEET along with Course code: KEM3, Topic and Assignment number.

Use unruled note, submit a clearer view of answers preferably in a single pdf file using adobe scan app (only one answer file for one assignment)

You may go through the related lecture session(s) before trying this!

Dead Line for submission of this assignment: 13th October, 2024 (SUN)

ARYABHATTA INSTITUTE OF MATHEMATICAL SCIENCES

INDIA

September 2024

MATHS OLYMPIAD - STD 9, 10, 11

ASSIGNMENT - 2

Hope you have studied the live class video before trying this worksheet !

Name:_____ Standard:_____

1 Algebra

1. Solve in real numbers: $\begin{cases} p+q-r=2 \\ p^2+r^2-q^2=-36 \\ pqr=462 \end{cases}$

2. Prove that the sum of the squares of five consecutive integers cannot be a perfect square.

   Give reasons.

2 Number Theory

3. Let L_k be the $L.C.M$ of the first k natural numbers.

   How many distinct numbers are there among the 20 numbers:

   That is, L_t where $1 \leq t \leq 20, t \in \mathbb{N} : L_1, L_2, L_3, \cdots, L_{19}, L_{20}$? Give reasons.

4. Find all primes of the form $\left(n^3+1\right)$ where $n \in \mathbb{N}$. Give reasons.

3 Geometry

5. $ABCD$ is a parallelogram and P is an interior point on CD such that $BC = BP = PD$.

   If $\angle APB = 72^\circ$ and $\angle ADP = 2x^\circ$, find the value of x. Give reasons.

6. Given a circle of radius 1 *unit* and AB is a chord of the circle with length 1 *unit*.

   If C is any point on the major segment, show that $AC^2 + BC^2 \leq 2\left(2+\sqrt{3}\right)$. Give reasons.

4 Combinatorics

7. The sum of the digits of a 4-digit number is 8 more than the product of its digits.

   One such example is 2024.

   How many such 4-digit numbers are there? Give reasons.

8. Find the number of solutions of the equation:

$$a + b + c = 12$$

where $a < 6, b < 6, c < 6$ are natural numbers. Give reasons.

KEEP ENJOYING MATHEMATICS!